Title	Bifurcation from Classical to Quantum distinguishability(3) Chaos and nonlinear dynamics in dissipative systems(including BEC and pattern formations), Chaos and Nonlinear Dynamics in Quantum-Mechanical and Macroscopic Systems)
Author(s)	IMAFUKU, Kentaro; IMAI, Hideki
Citation	物性研究 (2005), 84(3): 451-452
Issue Date	2005-06-20
URL	http://hdl.handle.net/2433/110219
Right	
Type	Departmental Bulletin Paper
Textversion	publisher

Bifurcation from Classical to Quantum distinguishability

Institute of Industrial Science, Univ. of Tokyo Kentaro IMAFUKU[1] , Hideki IMAI

古典二準位系（ビット）あるいは量子二準位系（キュービット）をアンシラとして、与えられた二つの熱浴の区別をすることを考える。そこで定義される古典あるいは量子判定能力の時間発展を調べ、量子論的判定能力が古典論的判定能力から分岐して出現することを示す。

1 Extended abstruct

Physical observable is not unique target on which we can speak of "dynamics". In fact, as to be seen here, it is also possible to discuss dynamics of "informatic ability" defined through some "informatic tasks" associated with physical (evoluting) states. Such dynamics can be highly nonlinear even in cases where the state evolution is linear itself.

In the present work, we consider, as an example of such "informatic ability", distinguishability of two thermalization processes which can be defined through the following two-party game[2] : *(1) Alice (with heatbath) and Bob (with ancilla) agree to use two (inverse) temperature* (β_0, β_1) *as parameters of the game. (2) Choosing one of the two temperature* β_j *with probability* π_j $(j = 0$ *or* $1)$*, Alice prepares heatbath whose state is equilibrium of the chosen temperature. (3) Getting information from the ancilla, which is interacting with the heatbath prepared by Alice, Bob guesses the temperature chosen by Alice. (4) If his guess is correct, Bob wins. Otherwise Alice wins.* In this game, the distinguishabillity of two thermalization processes can be defined as the probability of Bob's winning.

Here let us discuss what Bob can do when he has one qubit as ancilla. Let us suppose that the evolution of qubit affected by heatbath whose temperature β_j can be described as [3] $\rho_t^{(\beta_j)} = e^{-\mathcal{L}^{(\beta_j)}t}\rho_0$, where

$$\mathcal{L}^{(\beta)}\cdot = -\left(\sigma_- \ \cdot\ \sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-,\ \cdot\ \}\right) - e^{-\beta\omega}\left(\sigma_+ \ \cdot\ \sigma_- - \frac{1}{2}\{\sigma_-\sigma_+,\ \cdot\ \}\right). \tag{1}$$

[1] E-mail: imafuku@imailab.iis.u-tokyo.ac.jp

[2] Our purpose here is not to propose game being fair against malicious players, but just to explain situation we focus on. Therefore we are assuming that Alice and Bob always follow the rule of the game.

[3] As known well, this equation can be derived by simple model with some reasonable assumptions. Pay attention that we chose a representation so that the free Hamiltonian of the qubit is represented as $H_S = \frac{\omega}{2}\sigma_z$.

An optimal success probability of the distinction between $\rho_t^{(\beta_0)}$ (with probability π_0) and $\rho_t^{(\beta_1)}$ (with probability π_1) can be obtained as

$$\begin{aligned} p(\rho_t^{(\beta_0)}, \rho_t^{(\beta_1)}) &= \max_{E_0,E_1} \left[\pi_0 \mathrm{tr}\left(E_0\rho_t^{(\beta_0)}\right) + \pi_1 \mathrm{tr}\left(E_1\rho_t^{(\beta_1)}\right)\right] \\ &= \pi_0 + \sum \text{positive eigen value of} \left(\pi_1\rho_t^{(\beta_1)} - \pi_0\rho_t^{(\beta_0)}\right), \end{aligned} \tag{2}$$

where E_0 and E_1 ($E_0, E_1 \geq 0$ and $E_0 + E_1 = I$) are a POVM to distinuish the two states. Therefore, time evolution of the distinguishability of the two thermal processes are given as

$$P_{\text{qubit}}(t) = \max_{\rho_0} \left[p(\rho_t^{(\beta_0)}, \rho_t^{(\beta_1)})\right]. \tag{3}$$

Let us consider also a classical counterpart of the above. Since we are discussing thermalization process, the classical counterpart of the state evolution (1) would be a balance equation of a classical two levewl system. In fact, it is known that a physically relevant balance equation can be obtained by (1) with a restricted choice of initial states within diagonal state. [4] Thus we can introduce the classical counterpart of (3) as [5]

$$P_{\text{bit}}(t) = \max_{\rho_0 \in \Omega_D} \left[p(\rho_t^{(\beta_0)}, \rho_t^{(\beta_1)})\right]. \tag{4}$$

where Ω_D is a set of all diagonal states.

For simplicity, let us put $\pi_0 = \pi_1 = 1/2$ in the following. By simple observation, we can see the following facts: *(a) $P_{bit}(0) = P_{qubit}(0) = 1/2$, which simply means that what Bob can do at $t = 0$ is pure guess without additional hint. (b)$P_{bit}(t) \leq P_{qubit}(t)$ and $P_{bit}(t \to \infty) \to P_{qubit}(t \to \infty)$ hold as naturally expected. (c) The initial state which gives optimal distinguishability is always pure state. (d) Advantage of qubit ancilla with respect to classical two level ancilla appears at $t = t^* < \infty$ in a bifurcational way. (For given two temperature β_0 and β_1, $P_{bit}(t) = P_{qubit}(t)$ for $0 \leq t \leq t^*$ and $P_{bit}(t) < P_{qubit}(t)$ for $t^* < t$.)* It might be possible to construct a new cryptographic primitive based on the above facts.

Acknowledgments

This work was supported by the project on "Research and Development on Quantum Cryptography" of National Institute of Information and Communications Technology as part of the programme "Research and Development on Quantum Communication Technology" of the Ministry of Public Management, Home Affairs, Posts and Telecommunications of Japan.

[4] Diagonal state in a representation which diagonalizes σ_z. Notice that the time evolution of diagonal part of (1) has a closed form.

[5] Note that this definition automatically restricts POVM to diagonal projective measurement.